# Gamma Scalping PnL with Sticky-Strike Volatility Surface

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**ABSTRACT**

The standard argument for the Gamma scalping strategy on call options does not take into account the volatility surface, but only a smile/skew $K \mapsto \sigma_I(K)$. In this note, we give a short derivation of the profit-and-losses stochastic differential when a sticky-strike surface $(K,\tau) \mapsto \sigma_I(K,\tau)$ is introduced, in the spirit of Bergomi ([1], esp. §2.7.2 pp. 67-69). The result highlights the role of the forward total variance $\partial_\tau(\tau\sigma_I^2)$ of the surface in the profitability of the strategy, in contrast with the usual argument, emerging from a Vega correction on the time-decay of the call option value.

Let $\tilde{C} = \tilde{C}(t,T,K,S,\sigma)$ be the Black-Scholes call price, $\sigma_I = \sigma_I(K,\tau)$ is the sticky-strike implied volatility surface in the space (strike, time-to-maturity). An observed call price $C = C(t,T,K,S)$ is equated to its Black-Scholes price over the surface:

$$C(t,T,K,S) = \tilde{C}(t,T,K,S,\sigma_I(K,T-t))$$

The partial derivatives $\partial_S C, \partial_{SS} C$ (Delta, Gamma) are trivial due to the sticky-strike assumption, but $\partial_t C(t,T,K,S)$ (Theta) presents a Vega correction. Indeed:

$$\begin{aligned}
\partial_t C(t,T,K,S) &= \partial_t \tilde{C}(t,T,K,S,\sigma_I(K,T-t)) \\
&\quad - \partial_\tau \sigma_I(K,T-t)\partial_\sigma \tilde{C}(t,T,K,S,\sigma_I(K,T-t)) \\
\partial_S C(t,T,K,S) &= \partial_S \tilde{C}(t,T,K,S,\sigma_I(K,T-t)) \\
\partial_{SS} C(t,T,K,S) &= \partial_{SS} \tilde{C}(t,T,K,S,\sigma_I(K,T-t))
\end{aligned}$$

By Black-Scholes PDE ($r = 0$):

$$\begin{aligned}
&\partial_t \tilde{C}(t,T,K,S,\sigma_I(K,T-t)) \\
&= -\frac{1}{2}S^2\sigma_I^2(K,T-t)\partial_{SS}\tilde{C}(t,T,K,S,\sigma_I(K,T-t))
\end{aligned}$$

Now suppose that the $P$-dynamics of the underlying follow a driftless geometric Brownian motion $dS_t = \sigma_t S_t dW_t^P$ with realised volatility process $(\sigma_t)_{t\geq 0}$ satisfying standard assumptions for the Itô process to be well defined. The Delta-hedging profit-and-losses process $\Pi = (\Pi_t)_{t\geq 0}$ of a unit call with Black-Scholes Delta yields

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the differential:

$$
\begin{aligned}
d\Pi_t &= dC(t,T,K,S_t) - \partial_S \tilde{C}(t,T,K,S,\sigma_I(K,T-t))dS_t \\
&= \Big(\partial_t C(t,T,K,S_t) + \frac{1}{2}\sigma_t^2 S_t^2 \partial_{SS} C(t,T,K,S_t)\Big)dt \\
&+ \Big(\partial_S C(t,T,K,S_t) - \partial_S \tilde{C}(t,T,K,S_t,\sigma_I(K,T-t))\Big)\sigma_t S_t dW_t^P \\
&= \frac{1}{2} S_t^2 \partial_{SS}\tilde{C}(t,T,K,S_t,\sigma_I(K,T-t))\Big(\sigma_t^2 - \sigma_I^2(K,T-t) \\
&- 2\partial_\tau \sigma_I(K,T-t)\frac{\partial_\sigma \tilde{C}(t,T,K,S_t,\sigma_I(K,T-t))}{S_t^2 \partial_{SS}\tilde{C}(t,T,K,S_t,\sigma_I(K,T-t))}\Big)dt
\end{aligned}
$$

Since the Vega-weighted Gamma ratio is the time-weighted volatility, we get:

$$
\frac{\partial_\sigma \tilde{C}(t,T,K,S_t,\sigma_I(K,T-t))}{S_t^2 \partial_{SS}\tilde{C}(t,T,K,S,\sigma_I(K,T-t))} = (T-t)\sigma_I(K,T-t)
$$

So we conclude that the corrected scalping term is given by:

$$
\begin{aligned}
&\sigma_t^2 - \sigma_I^2(K,T-t) - 2(T-t)\partial_\tau \sigma_I(K,T-t)\sigma_I(K,T-t) \\
&= \sigma_t^2 - (\partial_\tau(\tau\sigma_I^2))(K,T-t)
\end{aligned}
$$

Hence we may want $\partial_\tau \sigma_I(K,T-t) < 0$ to profit from the correction. The profit-and-losses differential is:

$$
\boxed{d\Pi_t = \frac{1}{2} S_t^2 \partial_{SS}\tilde{C}(t,T,K,S_t,\sigma_I(K,T-t))\Big(\sigma_t^2 - (\partial_\tau(\tau\sigma_I^2))(K,T-t)\Big)dt}
$$

From $t$ to $t+\Delta t$ (small increment), freezing the Gamma term yields:

$$
\begin{aligned}
\Pi_{t+\Delta t} - \Pi_t &\approx \frac{1}{2} S_t^2 \partial_{SS}\tilde{C}(t,T,K,S_t,\sigma_I(K,T-t)) \cdot \\
&\Big(\int_t^{t+\Delta t} \sigma_u^2 du - \int_t^{t+\Delta t} (\partial_\tau(\tau\sigma_I^2))(K,T-u)du\Big) \\
&= \frac{1}{2} S_t^2 \partial_{SS}\tilde{C}(t,T,K,S_t,\sigma_I(K,T-t)) \cdot \\
&\Big(\int_t^{t+\Delta t} \sigma_u^2 du - \Big((T-t)\sigma_I^2(K,T-t) - (T-t-\Delta t)\sigma_I^2(K,T-t-\Delta t)\Big)\Big)
\end{aligned}
$$

In particular, the strategy is expected to profit from a calendar arbitrage on the surface if the total variance term in the brackets is $\leq 0$.

## References

[1] Bergomi, L. (2015). *Stochastic Volatility Modeling*. Chapman and Hall.