Addendum: "Note on the weak convergence of hyperplane α-quantile functionals and their continuity in the Skorokhod J1 topology"

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ARTICLE HISTORY
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(1) In the arguments for the proof of Th. 1.1, as is standard, we always assume that the variance-covariance matrix of a $\mathbb{R}^d$-Brownian motion with nontrivial covariance structure is nondegenerate (hence, strictly positive-definite). Indeed, if the variance-covariance matrix is degenerate, the finite-dimensional distributions of the process do not have a Gaussian density, hence the process is not a *bona fide* Brownian motion in $\mathbb{R}^d$, but takes values only in a linear subspace.

(2) If $\gamma = 0$, Th. 1.1 is automatically true since the choice forces $x \mapsto (M_{t,\alpha}(x), \tau_{M_{t,\alpha}}(x)) = (0,0)$, so that the case $\gamma = 0$ is implicitly ignored in the proofs. For clarity: Prop. 3.2, Prop. 3.6 and Le. 3.5 should read "$\gamma \in \mathbb{R}^d/\{0\}$" instead of "$\gamma \in \mathbb{R}^d$".